Theory of AC Josephson Effect in Superconducting Constrictions

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Abstract

We have developed a microscopic theory of ac Josephson effect in short ballistic superconducting constrictions with arbitrary electron transparency, and constrictions with diffusive electron transport. As applications of the theory we study smearing of the subgap current singularities by pair-breaking effects in the superconducting electrodes of the constriction, and the structure of these singularities in constrictions between the composite S/N electrodes with the proximity-induced gap in the normal layer.

It is quite surprising that despite the long history of Josephson junctions, the microscopic theory of the ac Josephson effect in junctions with arbitrary transparency is still absent. It is known that the basic mechanism of electron transport in such junctions at finite bias voltages is the process of multiple Andreev reflections (MAR) [1], but the first attempts at quantitative description of MAR based on the Green's function method [2–5] did not produce final results. Final progress in this direction has been made recently [6,7] but only for the case of fully transparent junctions. Quantitative description of MAR in junctions with arbitrary transparency is possible within the framework of the Bogolyubov-de Gennes (BdG) equations [8–11], but this approach does not allow to incorporate inelastic scattering and pair-breaking effects in the junction electrodes. This circumstance is crucial because dynamics of MAR makes finite-voltage electron transport in high-transparency junctions very sensitive to such scattering processes. The aim of the present work was to develop the theory of MAR in junctions with arbitrary transparency between the superconductors with general microscopic structure. The theory allows us to study, for example, the smearing of the MAR-related current singularities by pair-breaking scattering, and the ac Josephson effect between the normal conductors with the proximity-induced superconducting order parameter.

The basic model of a high-transparency Josephson junction is a short superconducting constriction (shorter than the coherence length ξ and elastic and inelastic scattering lengths of the electrodes) with a transparency D. The general approach to the description of such a constriction requires that we use the Green's function technique. In this technique, the constriction is described with the non-equilibrium quasiclassical Green's function $\check{G}$ which is 4x4 matrix consisting of 2x2 retarded, advanced, and Keldysh matrixes $\hat{G}^{R,A}$ and $\hat{G}$ [12,4]. To calculate the current we need to know only the asymmetric part $\check{J}$ of the Green's function $\check{J} = \check{G}(p_{zj}) - \check{G}(-p_{zj})$. Here p_z is momentum in the transport direction, and $j = 1, 2$ numbers the constriction electrodes. Solving the quasiclassical equations for $\check{G}$ inside the two electrodes and matching the solution across the constriction with the boundary conditions [4] one can show that the matrix $\check{J}(t, t')$ is continuous inside the constriction and is given

by the expression [4]:

$$\check{J} = 2D\check{G}_- * \check{G}_+ * (\check{\mathbf{1}} - D\check{G}_- * \check{G}_-)^{-1} = \check{G}_- * (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} - (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} * \check{G}_- \,. \qquad (1)$$

Here the product denoted by $*$ means the convolution with respect to the internal time variable, i.e., $\check{G}_\mu * \check{G}_\eta = \int dt_1 \check{G}_\mu(t,t_1)\check{G}_\eta(t_1,t')$, and we used the following notations: $\lambda = \sqrt{(1-D)/D}$, $\check{\mathbf{1}} = \check{1}(t-t')$, $\check{G}_\pm = (\check{G}_1 \pm \check{G}_2)/2$, $\check{G}_j(t,t') = \check{S}_j(t)\check{g}_j(t-t')\check{S}^*_j(t')$. In the last equation, $\check{g}_j(t) = \int \check{g}_j(\epsilon)\exp(-i\epsilon t)d\epsilon/(2\pi)$ is the equilibrium Green's function of jth superconductor: $\hat{g}_j(\epsilon) = [\hat{g}^R_j(\epsilon) - \hat{g}^A_j(\epsilon)]\tanh(\epsilon/2T)$, $\hat{g}^R_j(\epsilon) = g^R_j(\epsilon)\hat{\tau}_z + f^R_j(\epsilon)i\hat{\tau}_y = -[\hat{g}^A_j(\epsilon)]^*$, the matrix $\check{S}_j$ is : $\check{S}_j(t) = \exp[i\varphi_j(t)\check{\tau}_z/2]$ where $\varphi_j(t)$ is the phase of the order parameter in the jth electrode, and the phase difference $\varphi = \varphi_2 - \varphi_1$ is determined by the applied voltage $V(t)$: $\dot{\varphi}\ (t) = (2e/\hbar)V(t)$. Equation (1) gives the following expressions for the Keldysh component of $\check{J}$:

$$\hat{J} = (\hat{G}^R_2 * \hat{Q}_1 - \hat{Q}_1 * \hat{G}^A_2 - \hat{G}^R_1 * \hat{Q}_2 + \hat{Q}_2 * \hat{G}^A_1)/2 \qquad (2)$$

$$= \hat{J}^R * \hat{f}_+ - \hat{f}_+ * \hat{J}^A + (\hat{G}^R_2 * \hat{P}_1 - \hat{P}_1 * \hat{G}^A_2 - \hat{G}^R_1 * \hat{P}_2 + \hat{P}_2 * \hat{G}^A_1)/2 \qquad (3)$$

where $\hat{Q}_j = \hat{Q}^R * \hat{G}_j * \hat{Q}^A$, $\hat{Q}^\mu = (\hat{G}^\mu_+ + i\lambda\hat{\mathbf{1}})^{-1}$, $\hat{J}^\mu = \hat{G}^\mu_- * \hat{Q}^\mu - \hat{Q}^\mu * \hat{G}^\mu_-$, $\hat{P}_j = \hat{Q}^R * (\hat{G}^\mu_j * \hat{f}_- - \hat{f}_- * \hat{G}^A_j) * \hat{Q}^A$, $\hat{f}_\pm = (\hat{f}_1 \pm \hat{f}_2)/2$, and $\hat{f}_j(t,t') = \hat{S}_j(t)f(t-t')\hat{S}^*_j(t')$ is the matrix distribution function of the jth superconductor with $f(\tau) = \int \tanh(\epsilon/2T)\exp(-i\epsilon\tau)d\epsilon/(2\pi)$. The current in the constriction can be found from eq. (2) or (3):

$$I(t) = \frac{\pi}{2eR_0}\mathrm{Tr}\hat{\tau}_z\hat{J}(t,t)\,, \qquad (4)$$

where $R_0 = \pi\hbar/e^2$ for a single-mode constriction. For a constriction with large number of propagating electron modes and cross-section area $\mathcal{A}$, angular averaging over momentum direction should be taken into account in (4), and $R_0 = (2\pi/ep_{F1})^2\hbar^3/\mathcal{A}$, where $p_{F1} = \min p_{Fj}$.

Equations (2), (3) can be used to find the current not only in the ballistic constrictions but also in short diffusive constrictions, i.e. channels with large number of propagating modes and length d that satisfies the condition $l \ll d \ll \xi$, where l is elastic scattering

length. Indeed, in this case solution of the quasiclassical equations for Green's function gives for the matrix $\check{J}$ the expression $\check{J} = (2lv_{Fz}/dv_F)\ln(\check{G}_2 * \check{G}_1)$ [2], which reduces to the following form [13]:

$$\check{J} = \frac{4lv_{Fz}}{dv_F}\int_0^\infty \frac{d\lambda}{\sqrt{1+\lambda^2}}\left[\check{G}_- * (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} - (\check{G}_+ + i\lambda\check{\mathbf{1}})^{-1} * \check{G}_-\right]. \tag{5}$$

After the substitution: $\lambda = \sqrt{(1-D)/D}$, we get for the current

$$I(t) = \frac{\pi}{4eR_N}\int_0^1 \frac{dD}{D\sqrt{1-D}}\mathrm{Tr}\hat{\tau}_z\hat{J}(t,t;D)\,, \tag{6}$$

where $\hat{J}(t,t;D)$ is given by eq. (2), and R_N is the normal-state resistance of the channel. Equation (6) shows that similarly to the approach based on the BdG equations [11], in the general Green's function approach, the current in the diffusive superconducting constriction can be written as a sum of independent contributions from infinite number of ballistic propagating modes with the distribution of transparencies given by the Dorokhov's [14] density function $(\pi\hbar/2e^2R_N)/D\sqrt{1-D}$.

To calculate the current at arbitrary bias voltages across the constriction we should transform eqs. (2) and (3) further. Separating the normal-state contribution V/R_N, where $R_N = R_0/D$ is the normal-state junction resistance, we obtain the following symmetrized expression for the current:

$$I(t) = \frac{V(t)}{R_N} + \delta I_{12}(t) + \delta I_{21}(t)\,, \tag{7}$$

$$\delta I_{jk}(t) = \frac{\pi}{4eR_0}\mathrm{Tr}[\hat{\tau}_z(\hat{G}_j^R * \hat{q}_{jk} - \hat{q}_{jk} * \hat{G}_j^A)](t,t)\,, \tag{8}$$

where $\hat{q}_{jk} = \hat{q}_{jk}^R * \hat{g}_k * \hat{q}_{jk}^A$, $\hat{q}_{jk}^\mu = 2/(\hat{G}_j^\mu + \hat{g}_k^\mu + 2i\lambda\hat{\mathbf{1}})^{-1}$, and $\hat{g}_k = \hat{g}_k^R * f - f * \hat{g}_k^A$. In equation (8), all distribution functions correspond to zero potential, and in contrast to the previous definition, the function $\hat{G}_1^\mu$ and $\hat{G}_2^\mu$ are defined similarly, $\hat{G}_j^\mu = \hat{S}(t)\hat{g}_j^\mu(t-t')\hat{S}^*(t')$, where $\hat{S}(t) = \exp[i\varphi(t)\hat{\tau}_z/2]$. Different representations for the current can be obtained from eq. (8). We will use the representation in which the Fourier components $\hat{q}_{jk}^{R,A}(\epsilon,\epsilon')$ are expressed, respectively, through the matrices $\hat{\alpha}_{jk}^R(\epsilon,\epsilon')$ and $\hat{\alpha}_{jk}^A(\epsilon,\epsilon') = [\hat{\alpha}_{jk}^R(\epsilon',\epsilon)]^\dagger$, where, for example, $\hat{\alpha}_{jk}^R$ obeys the following equation:

$$(\hat{\rho}_k^R - \hat{\Gamma}_j^R * \hat{\eta}_k^R) * \hat{\alpha}_{jk}^R = r\hat{\mathbf{1}} + \hat{\Gamma}_j^R \,, \tag{9}$$

In this equation, $r = \sqrt{1-D}$ is the reflection amplitude of the constriction, and other notations are: $\rho_k^R(t,t') = \hat{\mathbf{1}} - r\Gamma_k^R(t-t')\hat{\tau}_x$, $\hat{\eta}_k^R(t,t') = -r\hat{\mathbf{1}} + \Gamma_k^R(t-t')\hat{\tau}_x$, $\Gamma_k^R(t) = \int \gamma_k^R(\epsilon)\exp(-i\epsilon t)d\epsilon/(2\pi)$, $\hat{\Gamma}_k^R(t,t') = \Gamma_k^R(t-t')\exp\{i\hat{\tau}_z[\varphi(t)+\varphi(t')]/2\}\hat{\tau}_x$, $\gamma_k^R(\epsilon) = f_k^R(\epsilon)/[g_k^R(\epsilon)+1]$, $\gamma_k^A(\epsilon) = \gamma_k^{R*}(\epsilon)$. As we will see later, $\gamma_k^R(\epsilon)$ has the meaning of amplitude of Andreev reflection at the interface between the constriction and the kth superconductor. In terms of $\hat{\alpha}_{jk}^{R(A)}$ eq. (8) reduces to the following form:

$$\delta I_{jk}(t) = \frac{1}{8eR_0}\int \frac{d\omega}{2\pi}\int d\epsilon J_{jk}(\epsilon, \epsilon-\omega)e^{-i\omega t} \,, \tag{10}$$

where

$$J_{jk}(\epsilon,\epsilon') = \mathrm{Tr} i\hat{\tau}_y[\gamma_k^R(\epsilon)W_k(\epsilon')\hat{\alpha}_{jk}^R(\epsilon,\epsilon') - \gamma_k^A(\epsilon')W_k(\epsilon)\hat{\alpha}_{jk}^A(\epsilon,\epsilon')]$$

$$+[1+\gamma_k^R(\epsilon)\gamma_k^A(\epsilon')]\mathrm{Tr}\hat{\tau}_z\int\frac{d\epsilon_1}{2\pi}W_k(\epsilon_1)\hat{\alpha}_{jk}^R(\epsilon,\epsilon_1)\hat{\alpha}_{jk}^A(\epsilon_1,\epsilon'), \;\; W_k(\epsilon) \equiv \left[1 - \left|\gamma_k^R(\epsilon)\right|^2\right] f(\epsilon).$$

Equations (10) and (7) show that the problem of finding the current in short ballistic superconducting constriction for arbitrary time-dependent bias voltage reduces to the problem of solving eq. (9), which is a Fredgholm integral equation (see, e.g., [15]). Similar procedure applied to (6) gives the current in diffusive constriction. It should be noted that this approach assumes that all frequencies (frequency of Josephson oscillations and typical frequency of voltage variations) are much smaller than the inverse of the time of electron motion through the constriction.

Equation (9) can be solved easily in the case of the dc bias voltage V, when the phase difference is: $\varphi(t) = \omega_J t + \varphi_0$, where $\omega_J = 2eV/\hbar$ is the Josephson oscillation frequency. In this case solution of eq. (9) can be written as a series , $\hat{\alpha}_{jk}(\epsilon,\epsilon') = 2\pi\sum_n \hat{\alpha}_{n(jk)}(\epsilon')\delta(\epsilon - \epsilon' - n\hbar\omega_J)$ (we omit the superscript R), in which the amplitudes $\hat{\alpha}_{n(jk)}$ are determined by a system of recurrence relations. To obtain these relations explicitly it is convenient to write the matrix $\hat{\alpha}_{jk}$ as

$$\hat{\alpha}_{jk} = \begin{pmatrix} b^+ & a^- \\ a^+ & b^- \end{pmatrix}_{(jk)} .$$

Equation (9) shows that the pairs of functions $a^s_{jk},\ b^s_{jk}$ with $s = \pm$ satisfy the same equations but corresponding to different polarity of the bias voltage. For dc voltage we get the following recurrence relations for the matrix elements of $\hat{\alpha}_{n(jk)}$:

$$a_{n+1} - \gamma_j(\epsilon_{2n+1})\gamma_k(\epsilon_{2n})a_n + r[\gamma_j(\epsilon_{2n+1})b_n - \gamma_k(\epsilon_{2n+2})b_{n+1}] = \gamma_j(\epsilon_1)\delta_{n0}, \tag{11}$$
$$c(\epsilon_{2n+1})b_{n+1} - d(\epsilon_{2n})b_n + c(\epsilon_{2n-1})b_{n-1} = -r\delta_{n0}\,,$$

where we used the simplified notations $a_n \equiv a^+_{n(jk)}(\epsilon)$ and $b_n \equiv b^+_{n(jk)}(\epsilon)$, and other notations are: $\epsilon_n = \epsilon + neV,\ c(\epsilon) = D\gamma_j(\epsilon)\gamma_k(\epsilon + eV)/[1 - \gamma_j^2(\epsilon)],\ d(\epsilon) = 1 - \gamma_k^2(\epsilon) + D\gamma_j^2(\epsilon + eV)/[1 - \gamma_j^2(\epsilon + eV)] + D\gamma_k^2(\epsilon)[1 - \gamma_j^2(\epsilon - eV)]$. The amplitudes $a_{n(jk)}$ and $b_{n(jk)}$ determine Fourier components of the current $I(t) = \sum_n I_n \exp(in\omega_J t)$, which according to eqs. (7) and (10) are given by the expression $I_n = V\delta_{n,0}/R_N + I_{n(12)} + I_{n(21)}$, where

$$I_{n(jk)} = \frac{1}{2eR_0}\int_{-\infty}^{\infty} d\epsilon W_k(\epsilon)\Phi_{n(jk)}(\epsilon), \tag{12}$$

$$\Phi_{n(jk)}(\epsilon) = -\gamma_k(\epsilon_{2n})a_{n(jk)}(\epsilon) - \gamma_k^*(\epsilon_{-2n})a^*_{-n(jk)}(\epsilon) +$$

$$\sum_m [1 + \gamma_k(\epsilon_{2m+2n})\gamma_k^*(\epsilon_{2m})][b_{n+m}b^*_m - a_{n+m}a^*_m]_{(jk)}(\epsilon), .$$

For a constriction between two BCS superconductors the recurrence relations (11) and equation (12) for the current reduce to the corresponding expressions that can be obtained from the BdG equations - see [9], where these expressions were derived for symmetric constriction. This means that for the purpose of description of the ac Josephson effect in a short constriction all information about the microscopic structure of a superconducting electrode of the constriction is contained in the function $\gamma^R(\epsilon)$ (introduced after eq. (9)) which has the meaning of the amplitude of Andreev reflection from this superconductor.

Equations (11) can be solved by standard methods [15] (see also [8]). Namely, it follows from (11) that $b_n(\epsilon) = b_0(\epsilon)\prod_{m=\pm 1}^{n} p_\pm(\epsilon_{2m})$, where the functions $p_\pm(\epsilon)$ are solutions of

the equations $p_\pm(\epsilon) = c(\epsilon_{\mp 1})/[d(\epsilon) - c(\epsilon_{\pm 1})p_\pm(\epsilon_{\pm 2})]$; the two signs ($\pm$) here correspond to $n \geq 1$ ($n \leq -1$), and $b_0(\epsilon) = r[d(\epsilon) - c(\epsilon_1)p_+(\epsilon) - c(\epsilon_{-1})p_-(\epsilon)]^{-1}$. These relations provide the basis for convenient numerical evaluation of the current. They also can be used to find analytical solutions in certain cases. In particular, in the low voltage limit, $eV \ll \Delta_j$, we get explicitly $p_\pm(\epsilon) = \kappa(\epsilon) - \sqrt{\kappa^2(\epsilon) - 1}$ where $\kappa(\epsilon) = \{[1-\gamma_1^2(\epsilon)][1 - \gamma_2^2(\epsilon)] + D\gamma_1^2(\epsilon) + D\gamma_2^2(\epsilon)\}/2D\gamma_1(\epsilon)\gamma_2(\epsilon)$, and $b_0(\epsilon) = r/2D\gamma_1(\epsilon)\gamma_2(\epsilon)\sqrt{\kappa^2(\epsilon) - 1}$. Thus, eqs. (11) and (12) enable us to find the current in the constriction between the superconductors with arbitrary quasiparticle spectrum. As was discussed above, this feature of our approach is very important, since all deviations of the quasiparticle spectrum from its "ideal" BCS form affect strongly the subharmonic gap structure and especially the low-voltage behavior of the current.

The quasiparticle spectrum of a superconductor can differ significantly from the BCS form, in particular, due to pair-breaking effects. One of the most important example of these effects is scattering on paramagnetic impurities – see, e.g., [16,17]. As a first application of our general theory we consider the ac Josephson effect in a constriction between two superconductors with paramagnetic impurities. The retarded Green's functions of such superconductors are determined by the relations [16]:

$$g^R = \frac{u}{\sqrt{u^2 - 1}} = uf^R \, , \quad \frac{\epsilon}{\Delta} = u[1 - \frac{\zeta}{\sqrt{1 - u^2}}] \, , \tag{13}$$

where ζ is the pair-breaking parameter, $\zeta = \hbar/\tau_s\Delta$, with τ_s being the spin-flip scattering time. Similar expressions for the Green's functions are valid also in the situations with other pair-breaking effects [16], for instance, for dirty thin superconducting film of thickness $d_S \ll \xi$ in magnetic field H parallel to the film. The Green's functions of such a film are given by eq. (13) with the pair-breaking parameter $\zeta = lv_F(eHd_S)^2/18\Delta$. Weak pair-breaking effects result in smearing of the BCS singularity in the quasiparticle spectrum and suppression of the superconducting energy gap to a reduced value $\Delta_g = \Delta(1 - \zeta^{2/3})^{3/2}$. The gap disappears completely at $\zeta \geq 1$. Figure 1 shows how these changes in the quasiparticle spectrum are reflected in the current-voltage ($I - V$) characteristic of the constriction. All

gap-related features in the $I-V$ characteristic are rapidly broadened at small ζ, and it becomes practically linear in the gapless regime $\zeta \geq 1$.

As another application of our theory we consider a constriction between two normal conductors in which superconductivity is induced by the proximity effect, i.e. an $S/NcN/S$ junction. Besides general interest to the proximity effect, the importance of this model is due to its relevance for realistic description of the high critical current tunnel junctions [18]. We consider a particular case of a thin dirty N layer of thickness $d_n \ll \xi_n$ with low transparency of the S/N interface $\langle D' \rangle \ll 1$. It is assumed that the resistance of the interface is still negligible in comparison to the constriction resistance. [19] The Green's functions in the N layer are given then by the first equation in (13) with $u = (\epsilon + i\gamma_b g_S^R)/i\gamma_b f_S^R$ – see, e.g., [18,21] and references therein. Here g_S^R and f_S^R are the Green's functions of the superconductor, and $\gamma_b/\hbar = \langle D' \rangle v_{Fn}/4d_n$ is the characteristic tunneling rate across the S/N interface which is assumed to be larger than electron-phonon inelastic scattering rate in the N layer. The energy gap Δ_g is induced in the N layer due to the proximity effect. If the S electrode of the structure is the BCS superconductor with energy gap Δ, the induced gap Δ_g is determined by the equation $\Delta_g = \Delta\gamma_b/[\sqrt{\Delta^2 - \Delta_g^2} + \gamma_b]$. Existence of the induced gap implies that there are two peaks in the density of states of the N layer, at energies Δ_g and Δ. This structure of the density of states results in a complex structure of the subharmonic gap singularities in the $I-V$ characteristics of the constriction. Example of such a structure is shown in Fig.2 for $\gamma_b/\Delta = 1$, when $\Delta_g \simeq 0.54\Delta$. We see that the most pronounced current singularities in this case are the subharmonic singularities at $eV = 2\Delta_g/n$ associated with the induced gap. Also visible are the singularities at "combination" energies $\Delta + \Delta_g$ and $(\Delta + \Delta_g)/2$.

In the case of fully transparent constriction with $D = 1$ we can obtain explicit analytical expressions for the current since the recurrence relations (11) can be solved directly in this case. In particular, in the low voltage limit $eV \ll \Delta_g f(\Delta_g)$ we get for symmetric constrictions:

$$I(t) = \frac{1}{2eR_N} \int d\epsilon \mathrm{Re} \frac{i \sin\varphi F_+(\epsilon, V) + 2u\sqrt{u^2-1}F_-(\epsilon, V)}{u^2(\epsilon) - \cos^2(\varphi/2)}, \tag{14}$$

$$F_\pm(\epsilon, V) = [F(\epsilon, V) \pm F(\epsilon, -V)]/2\,,$$

$$F(\epsilon, \pm V) = f(\epsilon) - \int_{\pm\Delta_g}^{\epsilon} d\epsilon' \frac{\partial f(\epsilon')}{\partial \epsilon'} \exp\left(-\int_{\epsilon}^{\epsilon'} \frac{dE}{eV} \ln\left|\gamma^2(E)\right|\right)\,.$$

These expressions imply that the current can be written as a sum of two contributions, one from the two discrete states inside the energy gap with energies $\epsilon_\pm = \pm\epsilon_\varphi$ determined by the equation $u(\epsilon_\varphi) = |\cos(\varphi/2)|$, and another contribution from the continuum of states above the gap. In contrast to constrictions between the BCS superconductors [22,23,7], in the general case of the non-BCS spectrum of quasiparticles both the discrete and continuum contributions to the current are significant. In particular, for the ballistic constriction in presence of pair-breaking effects one finds from eqs. (13) and (14) that the current I_d carried by the subgap states is: $I_d = \overline{I}\,|\sin(\varphi/2)|\left(1 - \zeta/\left|\sin^3(\varphi/2)\right|\right)\Theta\left(|\sin(\varphi/2)| - \zeta^{1/3}\right)$, where $\overline{I} = 2\mathrm{sgn}(V)\Delta_g f(\Delta_g)/eR_N$. At $\zeta = 0$ this expression reduces to the one found in [9]. The total current I in the constriction as a function of the (time-dependent) phase difference φ calculated from eqs. (13) and (14) is shown in Fig. 3. We see that even relatively small ζ has strong effect on the dynamic current-phase relation, suppressing the dc component of the current and making it more similar to the stationary current-phase relation.

In conclusion, we have developed the microscopic approach to the calculation of current in short ballistic and diffusive constrictions between the superconductors with arbitrary quasiparticle spectrum. We used the developed approach to study the time-dependent current and the dc current in constrictions between superconductors with the pair-breaking effects, and also between normal conductors with the proximity-induced superconductivity.

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[19] Similar model has been considered in [18], but the calculations of the current there were based on the phenomenological OBTK approach [20]. Although this approach is valid for fully transparent constrictions [7] it breaks down when the reflection coefficient is finite. The reason for this is that in [20] MAR is described in terms of quasiparticle energy distribution function. This does not allow to take into account quantum mechanical interference between different quasiparticle trajectories in the energy space which exist at finite reflection coefficient. Such interference is accounted for in the correct recurrence relations (11) for quasiparticle wave amplitudes.

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FIGURES

DC current-voltage characteristics of a short ballistic constriction with transparency D between the two superconductors with pair-breaking effects. The curves are shifted for clarity along the current axis and illustrate the smearing of the subharmonic gap structure with increasing strength of the pair-breaking. The values of the pair-breaking parameter ζ are (from bottom to top): $\zeta = 0.01$, 0.1, 0.3, 1.0. The upper curve with $\zeta = 1.0$ corresponds to the regime of gapless superconductivity.

DC current-voltage characteristics of a short symmetric $S/NcN/S$ constriction for different values of the constriction transparency D. The curves show the complex subharmonic gap structure associated with the two energy gaps: Δ in the S region, and proximity-induced gap Δ_g in the N region. Parameter γ_b characterizes the transparency of the S/N interfaces. For discussion see text.

Dynamic current-phase relation of a short symmetric constriction between superconductors with pair-breaking effects at low bias voltages, vanishing temperature, and different values of the pair-breaking parameter.